



Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

FURTHER MATHEMATICS

9231/41

Paper 4 Further Probability & Statistics

May/June 2024

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.



BLANK PAGE

- 1 The times taken by members of a large cycling club to complete a cross-country circuit have a normal distribution with mean μ minutes. The times taken, x minutes, are recorded for a random sample of 14 members of the club. The results are summarised as follows, where $\bar{x}$ is the sample mean.

$$\bar{x} = 42.8$$

$$\Sigma(x - \bar{x})^2 = 941.5$$

Find a 95% confidence interval for μ .

[4]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

- 3 A factory produces metal discs. The manager claims that the diameters of these discs have a median of 22.0 mm. The diameters, in mm, of a random sample of 12 discs produced by this factory are as follows.

22.4 20.9 22.8 21.5 23.2 22.9 23.9 21.7 19.8 23.6 22.6 23.0

- (a) Carry out a Wilcoxon signed-rank test, at the 10% significance level, to test whether there is any evidence against the manager's claim. [7]

[illegible]

- (b) State an assumption that is necessary for this test to be valid. [1]

.....

.....

- 4 The random variable Y is the sum of two independent observations of the random variable X . The probability generating function $G_Y(t)$ of Y is given by

$$G_Y(t) = \frac{t^2}{(4-3t)^4}.$$

- (a) Find $E(Y)$. [3]

[illegible]

- (b)** Write down an expression for the probability generating function of X . [1]

[illegible]

(c) Find $P(X = 4)$.

[3]

[illegible]

- 5 Two companies, P and Q , produce a certain type of paint brush. An independent examiner rates the quality of the brushes produced as poor, satisfactory or good. He takes a random sample of brushes from each company. The examiner's ratings are summarised in the table.

Company	Poor	Satisfactory	Good
P	18	43	64
Q	22	22	31

- (a)** Test, at the 5% significance level, whether quality of brushes is independent of company. [7]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** Compare the quality of the brushes produced by the two companies. [1]

[illegible]

[illegible]

(b) State an assumption that is necessary for this test to be valid. [1]

.....

.....

.....

The continuous random variable Y is defined by $Y = X^2$.

- (b) Find the probability density function of Y . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Find the exact value of the median of Y . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.